§16.2. ***Synopsis: Chapter Sixteen.***

The ease with which roots and powers of any number are found by logarithms is demonstrated. The method is extended to include any power in a G. P.

§16.2. ***Chapter Sixteen.*** [p.38.]

To find the root of any given number

With numbers from unity in continued proportion, such as 1, 3, 9, 27, 243, 729, 2187, that number which is nearest to one is called the root of all the terms succeeding[1]. Certainly the terms following are said to be powers of the same root: because they are constructed by the multiplication of the same root by itself, and in products from itself. The second of these from unity is called the square, the third the cube, the fourth the biquadratic: names are chosen for the remaining terms still further away following the distance that they maintain from unity. So the fifth term is called the fifth power; and the rest in the same manner. The characters of the numbers, however, by which those are recognised and distinguished in turn, are marked within circles [actually, brackets were used], as you see here [Table 16-1].

1	
3	(1)
9	(2)
27	(3)
81	(4)
243	(5)
729	(6)
2187	(7)

[Table 16-1]

39.] In those, 3 is the root of the square 9, the same 3 is the root of the cube 27, and of the biquadratic 81, and the same is the (7)th root of the seventh power (which is the furthest of these) 2187. Therefore, following those things which have been explained in the above chapter, the difference is taken of the logarithms of the proposed number and unity (that is, as the logarithm of unity shall be 0, the logarithm of the given number itself is taken) and divided by the number of intervals between the given number and unity; the quotient is the logarithm of the root sought. For if the square root is sought, half of the given logarithm is taken; if the cube root of the cube, a third; if of the power (7), the seventh part, etc. This fraction is the logarithm of the sought root. For if the given sixth power is 15625, of which the logarithm is 4,19382,00260,1611, and the (6)th root is sought. Therefore the sixth part of the logarithm found is taken, 0,69897,00043,3602, to which logarithm 5 corresponds. I assert that 5 is the root sought, as it appears along with these numbers, 1, 5, 25, 125, 625, 3125, 15625.

Let the given cube be 979: the cube root is sought. The logarithm 2,99078,26918,0314 is given of which a third is taken: 0,99692,75639,3438. In this example, and as in the above, the characteristic (which in Ch.4, we take to be the first place to the left) is to be carefully considered. And so, when the third part of the given first place cannot be taken, I put 0 for the first place of the quotient, for the characteristic of the same, which shows that the root sought is less than ten.

And since the root cannot be found from whole numbers, we consider the root sought to be multiplied by 10000, the logarithm of which 4,00000,0000,0000 is added to the third [of the logarithm] found; the total is 4,99692,7563934538, to which a little more than 99295 corresponds in the final Chiliad. Which through using differences and proportions, 99295,04202,067 is found with sufficient accuracy: that is 9929504202067, where nine places have been added on. But because the two final places 067 are greater than is correct, for which 047 must be substituted; as I cautioned in Chapter 11, and I have shown in Chapter 12, how close we are able to come to the root sought [using proportions].

And by this rule the root of any proposed power can be obtained, if not exactly, nevertheless approximately: and not only the root, but any other term of the same series; either between the given power and unity, or in a step further away. For, if the given sixth power is 16525 : the logarithm of the root is found: 0,69897,00043,3602. If I wish to know the fourth power of the same root, I multiply the logarithm of the root found by four, it is 2,79588,00173,4408, to which the number 625 corresponds. But if I want to know the ninth power of the same, I multiply by 9: 6,29073,00390,2418 is the logarithm of the same product, of which the characteristic 6 is revealed, with a number of 7 places sought, and hence that is larger than can be found among these Chiliads. Therefore, I reduce that first place, for 6 substituting 4, and the logarithm is 4,29073,00390,2418, which is sought in the twentieth Chiliad, to which the number 19531 nearly agrees. But since I wish to augment this number with a fractional part, which is deficient; and the proportional parts added from this Chiliad shall not indeed be exact; by the warning of Chapter 11, I take the complement of the given logarithm 0,70926,99609,7582, which is sought among the logarithms on page 22 (just as with the rule and example shown on p.23; [p.11-2 in this translation]), where the number itself beyond expectation is discovered, (and is placed directly opposite 512); this logarithm added to the given, gives 1,00000,00000,0000, to which the absolute number 100000 agrees (if we should increase the characteristic) [3]. Which on division by 8.8.8, the factors of the number 512, gives the final quotient 1953125. As you see here.

p.40.]

	5.(1)
	25.(2)
(The number) to be divided: 1.000000[000]	125.(3)
1st divisor: 8; 12500: 1st quotient.	625.(4)
2nd divisor: 8; 15625: 2nd quotient.	3125.(5)
3rd divisor: 8; 1953125: final quotient	15625.(6)
	78125.(7)
	390625.(8)
	1953125.(9)

[Table 16-2]

This final quotient is the ninth power of the root 5 previously found.

1	
1/3	(1)
1/9	(2)
1/27	(3)
1/81	(4)
1/243	(5)

[Table 16-3]

There are also the roots from fractions, the squares, cubes, etc., as you see here. And the roots of these can be found through logarithms. For the defective logarithm is taken of the given fraction, as we have shown in Chapter 10; of this half is the logarithm of the square root, a third the logarithm of the cube root, etc. For if the fraction $^{8}/_{27}$ is given, of which the logarithm is –0,52827,37771,6705, and the cube root of this fraction is sought. A third of the given logarithm is taken: – 0,17609,12590,5568. The number corresponding to this logarithm is the denominator of the fraction sought, of which the numerator is one. The denominator is 1$\underline{5}$, for the number indeed is less than 10, since the characteristic is 0. The root sought is therefore $^{10}/_{15}$. If the square root of the same fraction is sought, half of the same logarithm is taken – 0,26413,68885,8352, to which the denominator 1$\underline{8371173}$ for the root sought corresponds, of which the numerator is one. It can hence be shown that the numerator of the fraction must to be 1. Because the difference of the logarithms of the numerator and the denominator is the logarithm of the fraction, as we demonstrated in Chapter 10; therefore if the denominator is sought for the agreeing logarithm, it is unavoidable that the logarithm of the numerator shall be 0: that is, the numerator shall be 1. It cannot become otherwise, in order that the given difference of logarithms is maintained. But if we wish the same fractions to be expressed with other numbers, with any number to be taken for the numerator, or for the denominator: if a numerator has been assumed, of which the logarithm* is added to the logarithm of the fraction, the total will be the logarithm of the denominator. As with the cube root of the fraction $^{8}/_{27}$ the logarithm has been found – 0,17609,12590,5568. Let the numerator of the root sought be 12, to the logarithm of this, 1,07918,12460, that other logarithm added gives 1,25527,25051, to which 18 corresponds, which is the denominator; thus, the root sought is $^{12}/_{18}$. Now if the denominator is assumed, the logarithm of the fraction is to be taken from the logarithm of the given denominator, and the remainder is the logarithm of the numerator. For let the denominator of the same cube root sought be 36, for which the logarithm is 1,55630,25007,6729, from which the logarithm found of the root sought is taken away, 1,38021,12417,1161 will be left, the logarithm of the numerator 24, I assert the root sought to be $^{24}/_{36}$. For fractions that may be written one way or another, maintain the same value if the ratio of the numerator to the denominator is the same: and therefore if the logarithms of the numerators and denominators have the same difference, the terms are proportional from the definition of logarithms, and the fractions themselves are equal. Here you see an example.

	Logarithms	
8	0,90308,99869,9194	
27	1,43136,37641,5899	
$^{8}/_{27}$	–0,52827,37771,6705	
$l(3)^{8}/_{27}$	–0,17609,12590,5568	$^{10}/_{15}$
12	1,07918,12460,4762	
$l(3)^{8}/_{27}$	–0,17609,12590,5568	$^{12}/_{18}$
18	1,25527,25051,0330	

[Table 16-4]

*This style of speaking is less proper: as it should in fact be subtracting the negative logarithm from the positive logarithm, but since it is as through addition, I have considered speaking as everyone is accustomed.

.41.] We can also find the root of any given fraction if we look for the root of the numerator and the denominator, as the cube root of $^{8}/_{27}$ is $^{2}/_{3}$.

Indeed the homogeneous roots of the given terms are the terms of the root sought, as of the fraction $^{8}/_{27}$, the cube root is $^{2}/_{3}$ of which the numerator is the cube root of the given numerator 8; and the denominator 3 likewise is the cube root of 27.

If the square root of $^{729}/_{4096}$ is required:

		Logarithms	
Terms given	729	2,86272,75283,1797	2.A
	4096	3,61235,99479,6776	2.B
Terms sought	27	1,43136,37641,5898	1.A
	64	1,80617,99739,8388	1.B

The root (2) of the given fraction $^{729}/_{4096}$ is $^{27}/_{64}$.

[Table 16-5]

Concerning fractions, if we seek any other number placed in the same series with the root, the logarithm of the root is multiplied by the ratio of the separation from unity: the product is the logarithm of the homogeneous number sought. For, if the given fraction is $^{3125}/_{16807}$, I wish to know, out of four continued proportions between the given fraction and unity what is the third term from unity? But since there are four means, there are five intervals, and the fifth part of the logarithm of the given fraction is the logarithm of the root, which tripled is the logarithm of the number of the third proportional from unity. The whole operation you have lying here before your eyes.

		Logarithms	
Fraction given	$^{3125}/_{16807}$	3,49485,00216,8009	
		4,22549,02000,7129	
Logarithm of the fraction	*A*	– 0,73064,01783,9120.	5.B
Root (5) - - -	*B*	– 0,14612,80356,7824.	1.B
Logarithm of the cube	*C*	– 0,43838,41070,3472.	3.B
	D	– 1,02289,62497,4768.	7.B

[Table 16-6]

The cube, or the third from unity sought is $\frac{1000}{2744}$ or $\frac{125}{343}$.

If the seventh from unity is required, the logarithm of the root is multiplied by 7, the product *D* – 1,02289,62497,4768 is $\frac{10000000}{105413504}$ or $\frac{78125}{823543}$.

And this method gives us any number less than unity in the same series with the given term. Because if we wish to know another number in the same series continued above one, the same logarithm of the root is multiplied by the number of the intervals between that term required and unity: the product is the logarithm of the number sought. In fact, the same logarithm isequidistant from the logarithm of unity on both sides[3], because the logarithm of one lies between equal logarithms, since the logarithm of the number above unity is positive; and that below unity is indeed negative.

And as the equidistant logarithms on either side are written with the same places: thus the same terms transposed on either side express the same absolute number. As with these numbers which we have been most recently seeking: $^5/_7$ and $^7/_5$ are close on either side to unity. The cubes or the third powers from unity are $\frac{1000}{2744}$ and $\frac{2744}{1000}$, or $\frac{125}{343}$ and $\frac{343}{125}$. And unity is always the mean proportional between equidistant terms on either side for the same series. And in this way with numbers from unity in continued proportion, we are able to increase from fractions to whole numbers [i.e. improper fractions], and conversely with whole numbers to decrease to fractions.

§16.3. *Notes Chapter Sixteen*.

1 The Latin word *latus*, meaning *side* , is used by Briggs in various forms both here and in various places in the book: base number or common ratio would be perhaps equivalent modern terms for us to use , but we have decided always to use the word *root* , even if it is not always the most appropriate term in the modern sense. Thus, in the sequence of terms 1, a, a^2, a^3,... , a will be called the root of the sequence, for some number $a > 0$.

2 Thus, Briggs is saying: Take the number N/100 that I know approximately, but whose logarithm I know better, find the complement of this logarithm, which corresponds to a well-known number $\overline{N} = 5.12$; then, since $(N/100) \times \overline{N} = 10^5$, $N = 100 \times 10^5/\overline{N} = 10{,}000{,}000/5.12 = 1000{,}000{,}000/512$, etc.

3 i.e. a/b, 1, b/a have equidistant logarithms from 0, the logarithm of one, for positive numbers a and b. In modern terms, we can think of a G.P. as extending in both directions: ...,r^{-3}, r^{-2}, r^{-1}, 1, r, r^2, r^3,....

§16.4. Caput XVI. [p. 38.]

Dati cuiuscunque numeri latus quodlibet invenire.

In numeris ab unitate continue proportionalibus, ut 1. 3. 9. 27. 81. 243. 729. 2187. numerus ille qui est unitati proximus, dicitur latus omnium subsequentium. subsequentes vero dicuntur potestates eiusdem lateris: quia fiunt per multiplicationem eiusdem lateris in seipsum, & in factos a seipso. Horum secundus ab unitate dicitur Quadratus, tertius Cubus, quartus Biquadratus : reliqui vero remotiores, sortiuntur nomina, secundam illam quam obtinent ab unitate distantiam. ut quintus dicitur potestas quinta; reliquique eodem modo. Characteres vero, quibus isti cognoscuntur & distinguntur invicem,

[p.39.]

sunt numerorum notae circulis inclusae, ut hic vides.

1	
3	(1)
9	(2)
27	(3)
81	(4)
243	(5)
729	(6)
2187	(7)

In istis 3 est latus Quadrati 9. Idem 3 est latus Cubi 27, & Biquadrati 81, idemque est latus (7) potestatis septimae (quae est harum ultima) 2187. Idcirco, secundum ea quae superiori capite tradita sunt, sumatur differentia Logarithmorum numeri propositi & unitatis, (id est cum Logarithmus unitatis sit 0 sumatur ipse Logarithmus dati numeri) & per numerum intervallorum inter datum numerum & unitatem dividatur ; quotus erit Logarithmus lateris quaesiti. ut si Quadrati latus quaeratur, sumatur semissis dati Logarithmi, si cubi triens, si potestatis (7) pars septima, &c. haec pars erit Logarithmus quaesiti lateris. ut sit datus 15625 potestas sexta, huius Logarithmus 4,19382,00260,1611. Quaeritur latus (6), sumatur igitur inventi Logarithmi pars sexta 0,69897,00043,3602 huic Logarithmo respondet 5. aio 5 esse latus quaesitum. ut in his numeris apparet, 1. 5. 25. 125. 625. 3125. 15625.

Esto datus Cubus 979: quaeritur latus cubicum. Logarithmus dati est , 2,99078,26918,0314 huius triens sumendus est: 0,99692,75639,3438. in hoc exemplo, ut & in superiori, Characteristica (quam cap.4 accepimus esse primam notam versus sinistram) diligenter est respicienda. itaque cum datae primae notae pars tertia sumi nequit, pono 0 loco primo quoti, pro Characteristica eiusdem, quae ostendit latus quaesitum esse minus denario.

Et cum in integris latus nullum inveniri possit, cogitemus latus quaesitum multiplicari per 10000, cuius Logarithmus 4,00000,0000,0000 addatur trienti invento, totus erit 4,99692,7563934538, cui in ultima Chiliade respondet 99295, & amplius. quod per differentias & partem proportionalem satis accurate invenitur 99295,04202,067 id est 9<u>929504202067</u>, ubi Novenario partes sunt adiunctae. quod autem duae ultimae notae sunt maiores quam oportuit <u>067</u> : pro quibus substitui debent <u>047</u> , admonui cap.ii. & capite 12 ostendi, quomodo proxime possimus ad latus quaesitum accedere.

Atque ad hunc modum latus propositae cuiuscunque potestatis haberi poterit, si non vere, tamen proxime. & non tantum latus, sed alius quilibet eiusdem seriei; vel intra datam potestatem & unitatem, vel in gradu remotiore. ut si data sit potestas sexta 16525: Logarithmus lateris inventus est 0,69897,00043,3602. si cupiam scire quartam potestatem eiusdem lateris, multiplico inventum Logarithmum lateris per quaternarium, sit 2,79588,00173,4408, cui congruit numerus 625. sin scire velim potestatem nonan eiusdem multiplico per 9. factus 6,29073,00390,2418 erit eiusdem Logarithmus. cuius characteristica 6 ostendit, in numero quaesito esse septem notas; & idcirco, eum esse maiorem, quam ut inter istas Chiliadas reperiri possit. idcirco minuo illam primam notam, pro 6 substituens 4, eritque Logarithmus 4,29073,00390,2418 quaerendus in vicesima Chiliade, cui proxime congruit 19531. sed cum velim quod huic numero deest, in partibus supplere; et pars proportionalis addenda in hac Chiliade non sit adeo perfecta; per praeceptum cap. 11. capio dati Logarithmi complementum 0,70926,99609,7582, quod quaerendum est inter Logarithmos paginae 22 (prout praecepto & exemplo ostendit pag. 23.) ubi ipse numerus praeter spem invenitur, (eique e regione situs est 512) hic adiectus dato, dat 1,00000,00000,0000. cui numerus absolutus respondet (si Characteristicam augeamus) 100000. qui divisus per 8.8.8 factores numeri 512, dat quotum ultimum 1953125, ut hic vides.

[p.40.]

	5.(1)
	25.(2)
Dividendus: 1.000000[000]	125.(3)
divisor1^{us} : 8; 12500: 1st quotus primus.	625.(4)
divisor2^{us} : 8; 15625: 2nd quotus secundus.	3125.(5)
divisor3^{us} : 8; 1953125: quotus ultimus.	15625.(6)
	78125.(7)
	390625.(8)
	1953125.(9)

Hic quotus ultimus, est potestas nona lateris 5 prius inventi. Sunt etiam in partibus Latera, Quadrati, Cubi, &c. ut hic vides. & harum latera inveniti poterunt per Logarithmos. Sumatur enim datarum partium Logarithmus defectivus, ut

1	
1/3	(1)
1/9	(2)
1/27	(3)
1/81	(4)
1/243	(5)

capite 10 ostendimus. eius semissis erit Logarithmus lateris quadrati, triens cubici, &c. ut sint datae partes $^{8}/_{27}$ earum Logarithmus erit –0,52827,37771,6705, harum partium quaeritur latus cubicum. Sumendus est triens dati Logarithmi – 0,17609,12590,5568. numerus huic Logarithmo congruens erit denominator partium quaesitum, quarum numerator erit Unitas. denominator erit 15. est enim numerus minor denario, cum Characteristica sit 0. est igitur latus quaesitum $^{10}/_{15}$. Si earundem partium quaeratur latus quadraticum, sumendus est semissis dati Logarithmi – 0,26413,68885,8352, cui respondet 18371173 denominator lateris quaesiti, cuius numerator est 1, Quod autem numerator partium debeat esse 1, hinc potest esse manifestum. quod differentia Logarithmorum numeratoris & denominatoris est logarithmus partium, ut capite 10 ostendimus. Idcirco si quaeritur denominator huic Logaritho congruens, necesse est ut numeratoris Logarithmus sit 0:id est, numerator sit 1. alias fieri non poterit, ut illa data Logarithmorum differentia servetur. Quod si easdem partes alijs numeris exprimi velimus, poterit numerus quilibet sumi pro numeratore, vel denominatore. Si assumptus fuerit numerator, eius Logarithmo * addatur Logarithmus partium: totus erit Logarithmus denominatoris. ut lateris cubici partium $^{8}/_{27}$ Logarithmus inventus est –0,17609,12590,5568. esto lateris quaesiti numerator 12; huius Logarithmo 1,07918,12460, ille alter adiectus dat 1,25527,25051, cui congruit 18, qui erit denominator. ita ut latus quaesitum sit $^{12}/_{18}$. Sin assumptus fuerit denominator, Logarithmus partium auferendus est e Logarithmo dati denominatoris, eritque reliquus Logarithmus numeratoris. ut esto denominator eiusdem lateris cubici quaesiti 36, cui Logarithmus 1,55630,25007,6729, a quo auferatur lateris quaesiti inventus Logarithmus 1,38021,12417,1161 Logarithmus numeratoris 24, aio latus quaesitum esse $^{24}/_{36}$. Partes enim, utcunque scribantur, eundem servant valorem, si eadem sit ratio numeratoris ad denominatorem: & idcirco si Logarithmorum numeratoris & denominatoris eadem fuerit differentia , termini ex definitione Logarithmorum erunt proportionales, & partes ipsae erunt aequales. exemplum hic vides.

	Logarithmi.	
8	0,90308,99869,9194	
27	1,43136,37641,5899	
$^{8}/_{27}$	–0,52827,37771,6705	
$l(3)^{8}/_{27}$	–0,17609,12590,5568	$^{10}/_{15}$
12	1,07918,12460,4762	
$l(3)^{8}/_{27}$	–0,17609,12590,5568	$^{12}/_{18}$
18	1,25527,25051,0330	

** Locutio haec est minus propria: cum reuera sit subducendus Logarithmus Defectus ex Abundante, quod cum fiat per additionem. loquendum censui ut solet vulgus.

[p. 41.]

Poterimus etiam latus quodlibet partium datarum inveniri si quaerimus latera Numeratoris dati & Denominatoris. ut latus cubicum $^{8}/_{27}$ est $^{2}/_{3}$.

Sunt enim terminorum datorum latera homogenea, termini lateris quaesiti, ut partium $^{8}/_{27}$, latus cubicum est $^{2}/_{3}$ cuius numerator 2 est latus cubicum dati numeratoris 8. & denominator 3, est latus item cubicum nominatoris 27. Si quaeritur latus quadrati $^{729}/_{4096}$:

		Logarithmi.	
Termini dati.	729	2,86272,75283,1797	2.A
	4096	3,61235,99479,6776	2.B
Termini quaesiti.	27	1,43136,37641,5898	1.A
	64	1,80617,99739,8388	1.B

latus (2) datarum partium $^{729}/_{4096}$ is $^{27}/_{64}$.

Quod si in partibus alium quemlibet numerum quaeramus in eadem serie cum latere positum, multiplicetur Logarithmus lateris pro ratione distantiae ab unitate: factus erit Logarithmus homogeneus numeri quaesiti. ut si datae sint partes $^{3125}/_{16807}$ cupio scire e quatuor continue proportionalibus inter datas partes & unitatem quisnam sit tertius ab unitate, cum autem quatuor sint modij, erunt quinque intervalla; & pars quinta Logarithmi datarum partium, erit Logarithmus lateris, qui triplicatus erit Logarithmus numeri ab unitate tertij. totam operationem hic oculis subiectam habes.

		Logarithmi.	
Partes datae	$^{3125}/_{16807}$	3,49485,00216,8009	
		4,22549,02000,7129	
partium Logarith.	*A*	– 0,73064,01783,9120.	5.B
lateris (5) - - -	*B*	– 0,14612,80356,7824.	1.B
Logarithm. cubis.	*C*	– 0,43838,41070,3472.	3.B
	D	– 1,02289,62497,4768.	7.B

Si quaeritur septimus ab unitate multiplicatur Logarithmus lateris per 7, factus *D* – 1,02289,62497,4768 dabit $\frac{10000000}{105413504}$ or $\frac{78125}{823543}$.

Atque hic modus nobis dabit quemlibet numerum unitate minorem in eadem serie cum data. Quod si aliquem numerum scire velimus in eadem serie continuata supra unitatem, multiplicetur idem Logarithmus lateris, per numerum intervallorum inter quaesitum & unitatem: factus erit Logarithmum numeri quaesiti. adeo ut idem sit Logarithmus aequidistantium ab unitate utrinque. hoc autem interest inter aequales, quod numeri supra unitatem Logarithmus sit Abundam; infra unitatem vero defectivus.

Atque ut Logarithmi utrinque aequidistantium. ijsdem notis scribuntur: sic ijdem termini transpositi utrumque numerum absolutum exprimunt. ut in his ipsis quos nuperrime quaerebamus numeris. unitate utrinque proximi sunt $^5/_7$ & $^7/_5$ Cubi vel tertij ab unitate utrinque, sunt $\frac{1000}{2744}$ & $\frac{2744}{1000}$, vel $\frac{125}{343}$ & $\frac{343}{125}$. estque Unitas semper media proportionalis inter utrinque aequidistantes in eadem serie. Atque ad hunc modum in numeris ab unitate continue proportionalibus, poterimus a partibus ascendere ad integros, & contra ab integris descendere ad partes.