§23.1. ***Synopsis: Chapter Twenty - Three.***

In general, Briggs notes that the area *A* of an arbitrary triangle cannot be equal to its perimeter *P*, for the two quantities are related by the formula $A = r \times P/2$, where *r* is the radius of the inscribed circle. If the lengths of the sides of a triangle are known, then the area and hence the in-radius *r* can be found: hence, only in the circumstance $r = 2$ does $A = P$; otherwise, a triangle similar to the original is found by scaling the lengths proportionally, in order that $r = 2$ finally, and the result $A = P$ is again true.

A number of special triangles is then considered for which $A = P$. These include the equilateral triangle with sides of length $4\sqrt{3}$, for which *A* and *P* have the minimum value; a method for scaling isosceles triangles is given; and finally, if only the base and perimeter of a triangle are given, then geometrical properties of the ellipse are used to find the lengths of the sides of the triangle which set A = P.

§23.2. *Chapter Twenty Three*

To find a triangle, the area of which is equal to the perimeter,

(*if we are testing with numbers*).

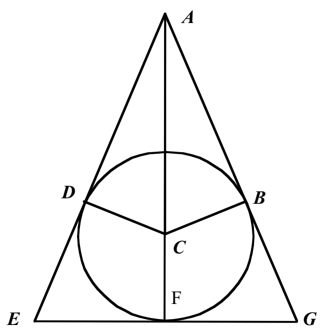
In general, no one consents that the perimeter of a triangle is equal to its area: as the perimeter and the area can be of different sizes, between which there can be no equality or ratio. But yet, if the total length comprising the measured perimeter arises as often as the same squared amount comprising the area of the triangle, then we can say in this particular case, that the area is equal to the perimeter of the same triangle.

If the area of the triangle is equal to the perimeter, the radius of the inscribed circle is 2, & conversely. For the area of the triangle is taken to be equal to the rectangle comprising [the product of] the radius of the inscribed circle and the semi-perimeter. As I have shown in section 3 of Ch. 18.

For a given triangle, to describe another similar triangle, of which the area is equal to the perimeter. The radius of the inscribed circle of the given triangle is found by section 3 of Ch. 18, & they become triangles in proportion: the radius given, the radius required 2; the sides given, & the sides required, being in proportion. The area of the triangle taken with sides in proportion is equal to the perimeter of the same triangle, because the radius of the inscribed circle is 2. For, let there be a given triangle of which the sides are 13, 14, 15. The radius of the inscribed circle is 4, & the proportionals are: 4: 2 ; 13:6_ ; 14: 7 ; 15: 7_ . I assert that the area of the triangle, of which the sides are 6_, 7, 7_ by section 1, Chapter 18, to be 21, is equal to the perimeter of the same triangle.

And by this manner, we can find the area of an equilateral triangle equal to the perimeter; for let each side be _.48, indeed the area is _.432, or 2078461, of which the altitude is 6, the triple of the radius of the inscribed circle, which altitude multiplied by half the length of the base _. 12, gives the area _.432. And this is the smallest perimeter and area of all the triangles found of this kind. [The length of side is 4√3.]

We can find any triangles with equal legs [i.e. isosceles] of this kind in this way: Let ABC [Fig. 23-1] be a right angled triangle with rational sides, by the method that was mentioned in Chapter 18, & let AF be equal to the sum of the hypotenuse AC and the side CB, & by proportion FG is required. As AB is to BC, so AF is to FG. FG and GB are equal, & triangle AEG of given sides with equal legs, & CF, CB, CD the equal radii of the inscribed circle. Let AB be 12; CB 5, and AC 13. AF is 18, and by proportion, FG, 7_, & AEG the triangle of the given equal length legs 19_, 19_, 15. & CF the radius 5 of the inscribed circle. And in order that a similar triangle can be constructed to this, of which the area is equal to the perimeter: the proportions are:-

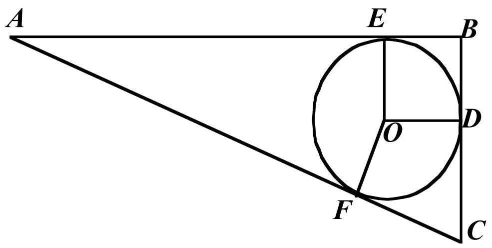


[Figure 23-1].

5, 2; 19_, $7^{4}/_{5}$; 15, 6; & the triangle taken with equal legs from the sides $7^{4}/_{5}$, $7^{4}/_{5}$, 6. Thus it has an area as well as a perimeter of $21^{3}/_{5}$ parts.

By the same method, if the sides are 5, 4, 3, ABC is the triangle with equal legs taken with the sides 10, 10, 12. The radius of the inscribed circle is 3. And for which the similar triangle is $6^{2}/_{3}$, $6^{2}/_{3}$, 8.

The right-angled isosceles triangle is the one and only of the kind, of which the sides are 4 +_.8, 4 +_.8, 4 + _.32. The area and the perimeter are both 12 +_.128, or 2331370849896.

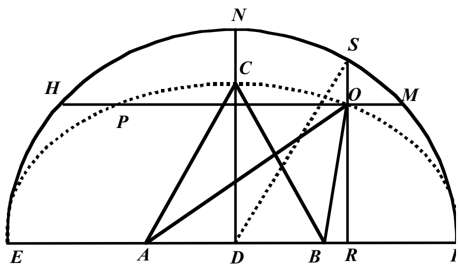


[Figure 23-2]

For scalene triangles of this kind [Fig. 23-2], if the perimeter is given, & and one of the angles is right, 4 [i.e. twice the in-radius] is taken away from the perimeter, the hypotenuse is half the remainder, because with a right-angled triangle *the diameter of the inscribed circle, is equal to the difference of the hypotenuse and the sum of the legs*, as is shown in Section 8, Chapter 19; & therefore, because the hypotenuse and the sum of the legs are given, the legs can be found by Section 6, Chapter 19.

.63.]

But if the perimeter and the base are given; first the altitude is sought by the proportional rule. For the base of the triangle is to half the perimeter, as the radius of the inscribed circle to the half of the altitude required. But with the altitude found, everything else can be completed in the manner explained in Chapter 20. Thus let the perimeter be 28, the base 8, and let ABC be a given isoperimetric isosceles triangle, constructed above the base AB, 8. So that AC and BC are both be 10. And with the radius ED 10, the semi-circular arc ENF is described, and the semi-elliptical arc ECF, then the altitude OR is sought [*i.e.* which must give $A = P$].



[Figure 23-3].

Proportions		*Logarithms*
Base AB - - - - - - - - - - - - -8		Comp. Ar. 9,09691,001
Radius of inscribed circle	2	0,30102,999
Given semi-perimeter	14	1,14612,804
Half the altitude OR	3_	(1)0,54406,804

[Table 23-1].

And drawing the perpendicular OR, 7, with HO parallel to the base, that cuts the arc of the ellipse in the point O, & AO, RO are drawn. I assert that the perimeter of the triangle AOB is 28 , is equal to the area of the same.

Proportions	*Logarithms*		
Sq. DC 84 Comp. Ar.	8,07572,071	_.$2666^{2}/_{3}$	51<u>63977795</u>
Sq. DN 100	2,00000,000	Sq. BO	55<u>01688871</u>
Sq. RO 49	1,69019,608	<u>BO</u>	7<u>4180111</u>
<u>Sq. RS $58^{1}/_{3}$</u>	(1)1,76591,697	Sq. AO	158<u>30644461</u>
Sq. DR $41^{2}/_{3}$		AO,	12<u>5819889</u>
BR. _. $41^{2}/_{3} - 4$			
[1]Sq. BR. _. bin. $57^{2}/_{3}$– _. $2666^{2}/_{3}$	AB 8		
<u>Sq. RO - - - - - - - - - 49</u>	AO 12<u>5819889</u>		
Sq. BO. _. bin. $106^{2}/_{3}$ – _. $2666^{2}/_{3}$	BO 7<u>4180111</u>		
Sq. AO._. bin. $106^{2}/_{3}$ + _. $2666^{2}/_{3}$	28<u>0000000</u> perimeter		

[Table 23-2].

To Be Noted. In triangle ABC, if 2 is taken from the length of the line CD, and CB, BD: CD − 2, XY become proportional. If XY is the larger of the two, AOB is the required triangle, with the inscribed circle, of which the radius is 2, and the area is equal to the given perimeter, otherwise, the area will be less than the perimeter.

§23.3. ***Note on Chapter Twenty Three***

[1] Note: In Table 23-2 _. bin. $57^{2}/_{3}$– _. $2666^{2}/_{3}$ means _. _. $57^{2}/_{3}$ - √$2666^{2}/_{3}$, or $57^{2}/_{3}$ - √$2666^{2}/_{3}$.

§23.4. **Caput XXIII.** [p.62.]

Triangulum invenire, cuius area (si numerum spectemus) aequetur Perimetro.

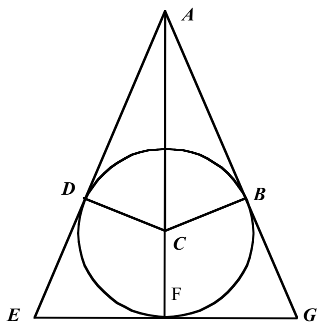
Lineam superficiei aequari posse nemo unquam probabit: cum sint heterogeneae magnitudines, inter quas nulla esse potest vel aequalitas vel ratio. veruntamen, si contigerit, mensuram longam toties contineri in perimetero, quoties eadem quadrata continetur in area trianguli; dicere poterimus, eius aream quodam modo aequari eiusdem Perimetero.

Si trianguli area aequetur perimetro, radius inscripti circuli est 2 & contra.. Est enim area trianguli aequalis rectangulo comprehenso a radio inscripti circuli & semiperimetro. ut ostendi sect.3.cap.18.

Dato triangulo, aliud simile describere, cuius area aequetur perimetro. Quaeratur radius circuli dato triangulo inscripti. per 3.sect.18.cap. & fiant, radius dati, radius quaesiti 2; latera dati, & latera quaesiti trianguli proportionalia. Trianguli a proportionalibus lateribus comprehensi area, erit aequalis perimetro eiusdem. quia radius inscripti circuli est 2. Ut esto datum triangulum cuius latera 13, 14, 15. radius circuli inscripti erit 4, & sunt: 4: 2 ; 13:6_ ; 14: 7 ; 15: 7_ proportionales. aio area trianguli, cuius latera sunt 6_, 7, 7_ per 1.sect.18.cap. esse 21, equalem perimetro eiusdem.

Atque hoc modo inveniemus aream trianguli aequilateri aequari perimetro; si unumquodque latus sit _.48, erit enim area _.432, vel 20<u>78461</u>, eius altitudo est 6, tripla radij circuli, quae ducta in basim dimidiatam _. 12, facit aream _.432. idem numerus proveniet, si latus perimeter _.48 triplicetur _.48 multiplicatum per _.9 facit _.432. Atque hoc est omnium huismodi triangulorum minimum.

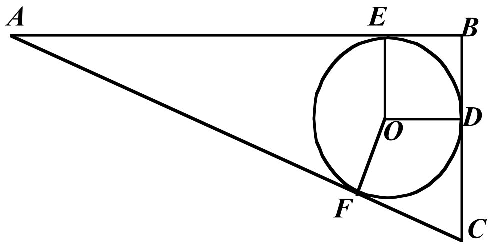
Triangula aequicrura huiusmodi quotlibet invenire poterimus hoc modo: fiat ABC triangulum rectangulum laterum rationalium, eo quo dictum est modo Cap. 18 & fiat AF aequalis hypotenusae AC & lateri CB, & proportionem quaeratur FG. Ut AB ad BC, sic AF ad FG. erunt FG, GB aequales, & AEG triangulum aequicrurum datorum laterum, & CF, CB, CD aequales radij inscripti circuli. Ut esto AB, 12; CB, 5; AC, 13. erit AF, 18, & per proportionem, FG, 7_, & AEG triangulum datorum laterum 19_, 19_, 15. & CF radius inscripti circuli 5. atque ut fiat triangulum huic simile, cuius area aequetur perimetro; erunt 5, 2; 19_, $7^{4}/_{5}$; 15, 6; proportionales, & triangulum equicrurum comprehensum a lateribus $7^{4}/_{5}$, $7^{4}/_{5}$, 6. habebit tam aream quam perimetrum partium $21^{3}/_{5}$.



[Figure 23-1].

Eodem modo si latera ABC sint 5.4.3, erit triangulum aequicrurum comprehensum a lateribus 10, 10, 12. radius circuli inscripti 3. & huic triangulo simile $6^2/_3$, $6^2/_3$, 8. cuius area & perimeter $21^1/_3$.

Triangulum isosceles rectangulum est unicum huius generis, cuius latera sunt 4 +_.8, 4 +_.8, 4 + _.32. area vero vel perimeter est 12 +_.128, or 2331370849896.

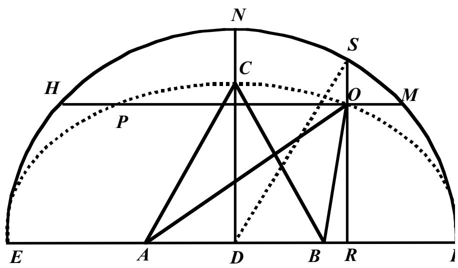


[Figure 23-2]

In triangulis huiusmodi scalenis, si data fuerit perimeter, & angulus unus sit rectus, auferantur 4 e perimetero, semissis reliqui erit Hypotenusa, quia in triangulo rectangulo *Diameter inscripti circuli, aequatur differentia Hypotenusa & summa crurum*, sect.8.cap.19; & idcirco, cum datae sint Hypotenusa [p.63.] & summa crurum, invenientur crura

Sin datae fuerint perimeter & basis; imprimis quaerenda est altitudo per proportionis regulam. Est enim basis trianguli, ad semiperimetrum; ut radius inscripti circuli, ad altitudinis quaesitae semissem. inventa autem altitudine, reliqua omnia peraguntur, eo quo dictum est modo, in Capite 20. Ut esto Perimetro 28, Basis 8, & fiat ABC triangulum isosceles isoperimetrum, dato, super basim AB, 8. ut AC, BC sint 10. Et radio ED 10, describatur semiperipheria circularis ENF, & semiperipheria Elleiptica ECF. deinde quaeratur Altitudo OR.

		Logarithmi
Proportiones		Comp.Ar.
Basis AB - - - - - - - - - - - - -8		9,09691,001
Radi circuli inscribendi	2	0,30102,999
Semiperimeter data	14	1,14612,804
semissis altitudinis OR	3_	(1)0,54406,804



Et ducta perpendiculari OR, 7, recta HO parallela basi, intersecet peripheriam Elleipticam in puncto O, & ducantur AO, RO. Aio perimetrum trianguli AOB 28 , equari areae eiusdem. sunt enim.

Proportiones	*Logarithmi.*		
Qu.DC 84 Comp. Ar.	8,07572,071	_.$2666^2/_3$	5163977795
Qu. DN 100	2,00000,000	Qu. BO	5501688871
Qu. RO 49	1,69019,608	BO	74180111
Qu. RS $58^1/_3$	(1)1,76591,697	Qu. AO	1583064461
Qu.. DR $41^2/_3$		AO,	125819889
BR. _. $41^2/_3$ – 4			
[1]Qu. BR. _. bin. $57^2/_3$– _. $2666^2/_3$		AB 8	
Qu. RO - - - - - - - - 49		AO 125819889	
Qu. BO. _. bin. $106^2/_3$ – _. $2666^2/_3$		BO 74180111	
Qu. AO._. bin. $106^2/_3$ + _. $2666^2/_3$		280000000 perim.	

Notandum. In triangulo ABC,si binarium auferatur e longitudine rectae CD, & fiant CB, BD: CD – 2, XY proportionales: si XY sit maior binario, poterit triangulo Quaesito AOB, circulus inscribi, cuius radius erit 2, eritque area perimetero data aequalis, alias, erit area minor perimetro.