§9.1. ***Synopsis: Chapter Nine.***

Upon evaluating the logarithms of 1, 2, 3, 5, 10, along with their associated products and quotients, Briggs now introduces a general scheme that will facilitate the evaluation of the logarithms of prime numbers. In particular, he demonstrates the method for primes $p < 100$, given that the logarithms of all primes less than p have been resolved already, as follows: it is possible to find three consecutive composite numbers a, b, c, that satisfy $b^2 = ac + 1$, one of which contains p as a factor, while the other primes present as factors are less than p. See Table 9 - 1. The quotient $b^2 / ac = 1 + 1/ac$ is formed, the r.h.s. of which is amenable to the continued means algorithm derived in the preceding chapter; consequently from the golden rule of proportionality, and the properties of logarithms, the logarithm of p is found.

§9.2. ***Chapter Nine*** [p.17.]

After finding the Logarithms of 1, 2, 3, 5, 10, and all the logarithms arrived at from these numbers by multiplication or division among themselves, according to Axioms 1 and 2 of Chapter 2: it remains that we seek the logarithms of the rest of the prime numbers, the investigation of which will be nearly the same as that before, though considerably easier. Indeed, two composite numbers are taken, of which the larger exceeds the smaller by one; and the first (of which the logarithm is sought) is a product from the given number reckoned either into itself or by a number of which the logarithm is given: the logarithm of the second [of the two numbers] assuredly is given. And in order that we can obtain two composite numbers of this kind more easily: that is taken first of which the logarithm is sought, with the two neighbouring numbers on both sides, which are always composite. [There are thus 3 consecutive numbers]. For the [original number in the] first position, either some multiple of itself is taken, with the two numbers on each side [also multiplied]; or from one nearby number [*i.e.* the prime with the unknown logarithm can be a factor of one of the end numbers], in which the product of the middle number taken in itself makes a number greater by one over the product from the outer numbers, by Prop.5, Book 2, Euclid[1]. Now the smaller product divides the larger, and by the continued means between the quotient and unity, the logarithm of the quotient is found, as we have shown before in Chapter 7. E.g. : The square of 7 (of which the logarithm is sought), reckoned with itself makes 2401; but the product of the nearby numbers on both sides, 48 & 50 is 2400. If this divides the larger, the quotient is 1$\underline{00041{}^{2}/_{3}}$,or 1$\underline{00041666666666666667}$ between which and one, the smallest of 44 continued means is 10000,00000,00000,00236,79824,90433,36405, of which the logarithm is: 0,00000,00000,00000,01028,40172,88387,29715, which doubled 44 times, or reckoned from 17592186044416, gives 0.00018,09183,45421,30 as the logarithm of the quotient 1$\underline{00041666666666666667}$, which added to the logarithm of the divisor, gives the logarithm of the dividend 2401, of which the fourth part is the logarithm of 7 sought. Here you see the whole operation of the method. All the reasoning is taken from the second chapter.

Factors	6	0,77815,12503,83643,63	
	8	0,90308,99869,91943,57	
Product	48	1,68124,12373,75587,20	
Factors	5	0,69897,00043,36018,81	
	10	1,00000,00000,00000,00	
Product	50	1,69897,00043,36018,81	
Factors		48	1,68124,12373,75587,20
		50	1,69897,00043,36018,81
Divisor Product		2400	3,38021,12417,11606,01
Quotient		1$\underline{00041{}^{2}/_{3}}$	0,00018,09183,45421,30
Dividend		2401	3,38039,21600,57027,31
Biquadratic		2401	3,38039,21600,57027,31
Quadratic		49	1,69019,60800,57027,31
Square root		7	0.84509,80400,14256,82

[Table 9-1]

And according to this method we can discover the logarithms of all prime numbers: among which these occur in the first place which are less than 100: which you can see on the following page placed in order, together with composite numbers, with the help of which most conveniently we arrive at those logarithms which we seek.

[p.18.]

Factors		*Product*	*To be divided*	*Divisor*	*Quotient*
	7.14	98			
11	11.9	99	9801	9800	100010204081632655306
	10.10	100			
	7.50	350			
13	13.27	351	194481	194480	1000008116883116883116 8
	16.22	352			
	17.26	442			
17	21.21	441	194481	194480	1000005141916906622783 8
	11.40	440			
	17.19	323			
19	18.18	324	104976	104975	10000095260776375327458 9
		325			
	13.30	390			
23	23.17	391	252881	252880	100000654107
	28.14	392			
	23.24	552			
29	29.19	551	303601	303600	1000003293807
	55.10	550			
	19.18	342			
31	31.11	341	116281	116280	10000085999931
	17.20	340			
	37.27	999			
37	100.10	1000	1000000	999999	1000001000001000001000
	91.11	1001			
	54.22	1188			
41	41.29	1189	1413721	1413720	10000007
	70.17	1190			
	90.11	990			
43	43.23	989	978121	978120	10000012
	52.19	988			
	47.21	987			
47	52.19	988	976144	976143	10000012
	43.23	989			
	43.16	688			
53	53.13	689	474721	474720	10000002
	23.30	690			
	66.17	1122			
59	59.19	1121	1256641	1256640	
	56.20	1120			
	43.27	1161			
61	29.40	1160	1345600	13455599	
	61.19	1159			
	67.23	1541			
67	77.20	1540	2371600	2371599	
	57.27	1539			
	71.29	2059			
71	49.42	2058	4235364	4235363	
	121.17	2057			
	54.23	1242			
73	73.17	1241	1540081	1540080	
	31.40	1240			
	79.18	1422			
79	49.29	1421	2019241		
	71.20	1420			
	49.22	1078			
83	83.13	1079	1164241		
	27.40	1080			
	55.21	1155			
89	34.34	1156	1336336		
	89.13	1157			
	97.11	1067			
97	82.13	1066	1136356		

71.15	1065			

[Table 9-2]

[p.19.] Another example illustrates the same. Let the logarithm of 17 be sought.

Factors	*Product*	*dividend*	*Divisor*	*Quotient*
7.17	119 *A*			
17.10.12	120 *B*	14400	14399	10000,69449,26731,02298,77074,79686
11.11	121 *C*			

[Table 9-3]

The Logarithm of the quotient found by continued means 0,00003,01603,86266,27349, which taken from the logarithm of the dividend 14400 leaves the Logarithm of the divisor 14339. And for the same reason, the logarithm of the number *C*, taken from the logarithm of the divisor, leaves the logarithm of the number *A*, which is equal to the logarithms making 7 and 17. All the working you see placed here before your eyes.

			Logarithms
Latus	Factors	10	1,00000,00000,00000,0000
		12	1,07918,12460,47624,82808
	Product	120	2,07918,12460,47624,82808
	Squared	14400	4,15836,24920,95249,65616
Latus		11	1,04139,26851,15226,17055
Squared		121	2,08278,53703,16452,34110
	Dividend	14400	4,15836,24920,95249,65616
	Quotient	100009644926	0,00003,01603,86266,27349
	Product of the Divisor - - - - - - - -14339		4,15833,23317,08983,38267
	Factors - - - - - - - - - - - - - - - - -	121	2,08278,53703,16452,34111
	Product	119	2,07554,69613,92531,04157
	Factors	7	0,84509,80400,14256,83080
		17	1,23044,89213,78274,21077

[Table 9-4]

§9.3. *Notes On Chapter Nine.*

[1] Briggs does not enlighten us on how he found his consecutive numbers: presumably he had a table of the prime factors of the integers available. In these sentences, he tells us that the unknown prime can be a factor of the middle number, or in one of the end numbers. We give another example in modern notation:

It is required to find the log of 79, the logs of all primes < 79 having been found already, consider the products from Table 9-2 : –

$a = 79 \times 18 = 1422$,

$b = 49 \times 29 = 1421$,

and $c = 71 \times 20 = 1420$.

From $b^2 - 1 = ac = 2019240$ the fraction $b^2/ac = 1 + 1/ac = 1^1/_{2019240} = 1.00000049523583..$ is amenable to the successive means method – and hence log 79 can be found.

§9.4. Caput IX [p.17]

Post inventos Logarithmos horum numerorum 1.2.3.5.10. , & per 1.& 2. ax. cap.2. omnium ex mutua multiplicatione vel divisione provenientium: restat ut reliquorum primorum Logarithmos quaeramus. quorum investigatio eadem fere erit quae antae, aliquanto tamen facilior. Sumantur enim duo numeri compositi, quorum maior minorem unitate tantum superat; & sit primus, factus a dato (cuius Logarithmus quaeritur) ducto vel in seipsum, vel in numerum cuius Logarithmus datus erit: secundi vero Logarithmus sit datus. Atque ut huiusmodi duos compositos facilius consequamur, sumatur primus ille cuius Logarithmus quaeritur, cum duobus utrinque proximis, qui semper sunt compositi. vel loco primi, sumatur multipl[ic]us eiusdem, cum duobus utrinque, vel ex altera parte proximis, horum medius in seipsum ductus, facit numerum unitate superantem factum a circumpositis, per 5.p.2.lib.Eucl. Factus autem minor maiorem dividat, & per continue medios inter quotum & Unitatem, quaeratur Logarithmus quoti, ut antea Capite 7. ostendimus. Exempli gratia: Quadratus septenarii (cuius Logarithmus quaeritur) 49 in seipsum ductus facit 2401; factus autem ab utrinque proximis 48 & 50 est 2400. hic si majoram dividat, quotus erit $100041\frac{2}{3}$ vel 100041666666666666667, interquem & Unitatem, minimus quadraginta quatuor continue Mediorum est 10000,00000,00000,00236,79824,90433,36405, cuius Logarithmus 0,00000,00000,00000,01028,40172,88387,29715, qui quadragies quater duplicatus, vel ductus in 17592186044416, dabit 0.00018,09183,45421,30 Logarithmum quoti 100041666666666666667, qui additus Logarithmo Divisoris 2400, dabit Logarithmum Divisi 2401, cuius pars quarta est Logarithmus Septenarii quaesitus. totius operationis modum hic vides. ratio omnium e secundo capite petenda.

Factores	6	0,77815,12503,83643,63
	8	0,90308,99869,91943,57
Factus	48	1,68124,12373,75587,20
Factores	5	0,69897,00043,36018,81
	10	1,00000,00000,00000,00
Factus	50	1,69897,00043,36018,81

Factores	48	1,68124,12373,75587,20
	50	1,69897,00043,36018,81
Divisor factus	2400	3,38021,12417,11606,01
Quotus	$100041\frac{2}{3}$	0,00018,09183,45421,30
Divisus	2401	3,38039,21600,57027,31
Biquadratus	2401	3,38039,21600,57027,31
Quadratus	49	1,69019,60800,57027,31
Latus 7		0.84509,80400,14256,82

[p.18]

Atque ad hunc modum invenire poterimus Logarithmos omnium primorum numerorum: inter quos primo loco occurrunt illi, qui sunt centenario minores: quos vides pagina sequente suo ordino positos, una cum numeris compositis, quorum ope commodissime illos quos quaerimus Logarithmos assequemur

Factores		*Facti*	*Dividendi*	*Divisores*	*Quoti.*
	7.14	98			
11	11.9	99	9801	9800	100010204081632655306
	10.10	100			
	7.50	350			
13	13.27	351	194481	194480	100000811688311688311168
	16.22	352			
	17.26	442			
17	21.21	441	194481	194480	100000514191690662278338
	11.40	440			
	17.19	323			
19	18.18	324	104976	104975	100000952607763753274589
		325			

23	13.30 23.17 28.14	390 391 392	252881	252880	100000654107
29	23.24 29.19 55.10	552 551 550	303601	303600	1000003293807
31	19.18 31.11 17.20	342 341 340	116281	116280	10000085999931
37	37.27 100.10 91.11	999 1000 1001	1000000	999999	1000001000001000001000
41	54.22 41.29 70.17	1188 1189 1190	1413721	1413720	10000007
43	90.11 43.23 52.19	990 989 988	978121	978120	10000012
47	47.21 52.19 43.23	987 988 989	976144	976143	10000012
53	43.16 53.13 23.30	688 689 690	474721	474720	10000002
59	66.17 59.19 56.20	1122 1121 1120	1256641	1256640	
61	43.27 29.40 61.19	1161 1160 1159	1345600	13455599	
67	67.23 77.20 57.27	1541 1540 1539	2371600	2371599	
71	71.29 49.42 121.17	2059 2058 2057	4235364	4235363	
73	54.23 73.17 31.40	1242 1241 1240	1540081	1540080	
79	79.18 49.29 71.20	1422 1421 1420	2019241		
83	49.22 83.13 27.40	1078 1079 1080	1164241		
89	55.21 34.34 89.13	1155 1156 1157	1336336		
97	97.11 82.13 71.15	1067 1066 1065	1136356		

[p.19.]

Alterum exemplum idem illustrabit. Sit querendus Logarithmum numeri 17.

Factores	*Factus*	*Dividendus*	*Divisor*	*Quotus*
7.17	119 *A*			
17.10.12	120 *B*	14400	14399	10000,69449,26731,02298,77074,79686
11.11	121 *C*			

Logarithmus Quoti per continue medios inventos 0,00003,01603,86266,27349, qui ablatus e Logaritho dividendi 14400 relinquit Logarithmum divisoris 14339, & eidem de causa, Logarithmus numeri c, ablatus e Logarithmo Divisoris, relinquit Logarithmum numeri A, qui aequatur Logarithmis facientium 7 & 17. totam operationem ob oculos positam hic vides.

			Logarithmi.
Latus	Factores	10	1,00000,00000,00000,0000
		12	1,07918,12460,47624,82808
	Factus	120	2,07918,12460,47624,82808
	Quadratus	14400	4,15836,24920,95249,65616
Latus		11	1,04139,26851,15226,17055
Quadratus		121	2,08278,53703,16452,34110
	Divisus	14400	4,15836,24920,95249,65616
	Quotus	100009644926	0,00003,01603,86266,27349
	Factus Divisor - - - - - - - -	14339	4,15833,23317,08983,38267
	Factores - - - - - - - - - - - - - - - - - -	121	2,08278,53703,16452,34111
	Factus	119	2,07554,69613,92531,04157
	Factores	7	0,84509,80400,14256,83080
		17	1,23044,89213,78274,21077